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SELF-FOCUSSED AND SELF-TRAPPED BEAMS IN NONLINEAR OPTICS

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Robert D. Small

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Mohamed N. Y. Anwar and Robert D. Small

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ABSTRACT

Self-focussing and self-trapping of light rays are studied in a classical two-dimensional nonlinear model both in the geometrical optics approximation and when retaining higher dispersion terms. The paraxial ray approximation of Akhmanov (1) that leads to the nonlinear Schrödinger equation is specifically avoided. It is found that in the geometrical optics approximation, focussing must eventually occur once convergence of rays begins. When both diffraction and nonlinearity are present, however, focussing may or may not occur.

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SIGNIFICANCE AND EXPLANATION

In the classical theory of light, rays are straight except for the smearing effect of diffraction that occurs when there is a sudden change in intensity such as would occur when light is partially blocked by an obstacle. Then light diffuses to some extent into the shadow zone so that the boundary between light and dark is not sharp. An additional effect that was discovered when lasers were developed is that in some media, when the light intensity is sufficiently great, the behavior of light depends on its brightness. This effect is called nonlinearity because the governing equations become nonlinear and one of the consequences is that beams of light can be caused to focus, giving such great intensities as to damage the crystal medium. Short of this the rays can become trapped so that they remain in a tight bundle and never spread. This paper studies the process of self-focussing and self-trapping on a simple model and, in particular, refutes the assertion of Akhmanov (1) that, for this model, once rays begin to converge, a focus must result. It turns out that the smearing effect of diffraction can prevent focussing although it need not do so.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the authors of this report.

SELF-FOCUSSED AND SELF-TRAPPED BEAMS IN NONLINEAR OPTICS

Mohamed N. Y. Anwar and Robert D. Small

Introduction

Light rays in certain nonlinear media converge solely due to the presence of the beams themselves. The property responsible for self-focussing and self-trapping of beams is that the phase velocity of the waves decreases with increasing amplitude. A disturbance that has a distribution of amplitude along an initially flat wave front will be retarded at its most intense point, as shown in Figure 1a, and rays will bend inwards. The convergence of rays increases the amplitude, enhancing the nonlinear effect, resulting in greater convergence until a singularity occurs. Rays bend not only due to nonlinearity but also due to diffraction which resists sudden changes in amplitude causing rays to bend outwards to disperse the energy. When focussing begins diffraction increases to oppose it. Diffraction can, in fact, restore a focussing beam to its original state as in Figure 1b. In this paper we demonstrate that the effect of nonlinearity alone causes focussing but that the combined effect of nonlinearity and diffraction can lead either to a focus or not, depending on the initial configuration of the rays.

The analysis for typical nonlinear media is very difficult since the media are, in general, anisotropic and also require a quantum mechanical description. The present work studies the self-focussing effect of a classical isotropic model in two dimensions. This has been attempted before, for example (1) and (2) but, for the most part, previous work in both geometrical and dispersive approximations has depended upon a further approximation where rays must be nearly parallel to the axis of the beam and in the dispersive case the governing equation is the nonlinear Schrödinger equation (1). In the region of any focus, however, the ray angles necessarily become large and violate that approximation. We shall show by explicit solutions that in the absence of diffraction, nonlinearity causes focussing and that when diffraction is present it can prevent focussing. This is

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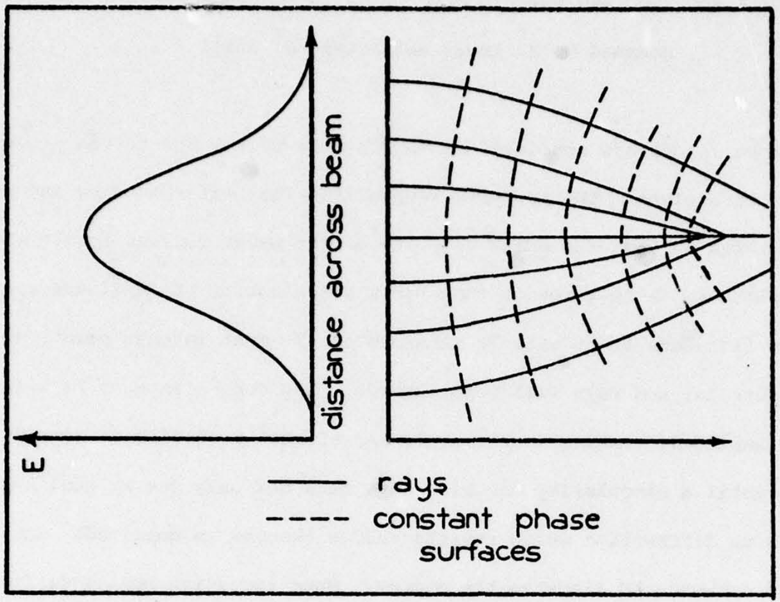

Figure 1a. Focussing Due to Nonlinearity

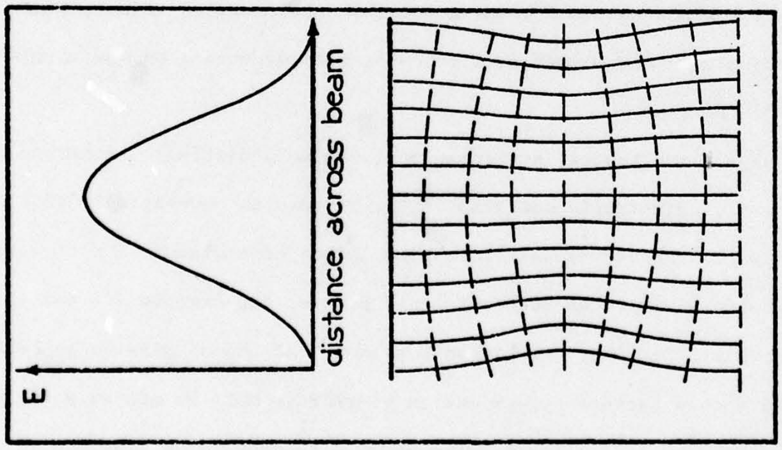

Figure 1b. Self-Trapping Due to Nonlinearity and Dispersion

in contradiction to Akhmanov et al. (1) who claim that diffraction cannot prevent the focussing that occurs in its absence.

1. The Governing Equations

The electromagnetic waves in our model are described by the macroscopic Maxwell equations in vacuum with the mobile particles of the medium entering by the source terms. The dynamic properties of the medium are fixed by considering it a continuum of electric dipoles, the negative particle feeling a force drawing it to a stationary positive core. Newton's law for the mobile negative particle completes the system of equations. These coupled equations accommodate two-dimensional solutions that represent linearly polarized waves with all vectors pointing in the z-direction. The magnitudes of the electromagnetic field intensity, E, and the polarization, P, obey the following equations, the first expressing Maxwell's equations and the second, Newton's law.

$$[1] \qquad \frac{\partial^2 E}{\partial t^2} - c^2 \left(\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} \right) = - \frac{1}{\varepsilon_0} \frac{\partial^2 P}{\partial t^2}$$

$$[2] \qquad \frac{\partial^2 P}{\partial t^2} + \omega_0^2 P - \alpha P^3 = \varepsilon_0 \, \omega_p^2 \, E$$

The constants are the conventional ones in MKS units and ω_p, ω_0 and α are determined by the medium. The cubic nonlinearity is chosen since it is the first non-linear term in a Taylor series for the isotropic restoring force and we take α to be small.

Since [1] and [2] are almost linear, solutions very nearly take the form of sinusoids with arguments linear in x,y and t. Thus we make the substitution

$$[3] \qquad \begin{aligned} E &= a(x,y,t) \cos \theta(x,y,t) \\ P &= b(x,y,t) \cos \theta(x,y,t) + \bar{b}(x,y,t) \sin \theta(x,y,t). \end{aligned}$$

The envelope amplitudes a, b and $\bar{b}$ are in general more slowly varying than the trigonometric functions of θ and since θ depends on x, y and t almost linearly we can define the wave number vector $\vec{\kappa} = (\kappa_1, \kappa_2)$ and the frequency ω by

$$\kappa_1(x,y,t) = \frac{\partial\theta}{\partial x} , \qquad \kappa_2(x,y,t) = \frac{\partial\theta}{\partial y} ,$$

[4]

$$\omega(x,y,t) = -\frac{\partial\theta}{\partial t} .$$

These scalars will be slowly varying as well. The result of substituting [3] into [1] and [2] and equating coefficients of $\cos\theta$ and $\sin\theta$, retaining one order of smallness in α, is the set of equations

[5]

$$a_{tt} - c^2(a_{xx} + a_{yy}) + (c^2\kappa^2 - \omega^2)a = - \varepsilon_0^{-1}(b_{tt} - \omega^2 b - 2\omega\bar{b}_t - \omega_t\bar{b})$$

$$2\omega a_t + \omega_t a + c^2(2\kappa_1 a_x + 2\kappa_2 a_y + \kappa_{1x}a + \kappa_{2y}a) = - \varepsilon_0^{-1}(2\omega b_t + \omega_t b + \bar{b}_{tt} - \omega^2 b)$$

$$b_{tt} + (\omega_0^2 - \omega^2)b - 2\omega\bar{b}_t - \omega_t\bar{b} - \frac{3}{4}\alpha b(b^2 + \bar{b}^2) = \varepsilon_0\omega_p^2 a$$

$$2\omega b_t + \omega_t b + \bar{b}_{tt} + (\omega_0^2 - \omega^2)\bar{b} - \frac{3}{4}\alpha\bar{b}(b^2 + \bar{b}^2) = 0 .$$

We have used $\kappa = |\vec{\kappa}|$ and subscripts to indicate partial differentiation. In addition the consistency relations from [4] are

[6] $$\kappa_{1y} = \kappa_{2x} , \quad \kappa_{1t} + \omega_x = 0 , \quad \kappa_{2t} + \omega_y = 0 .$$

We now consider only time independent envelopes. Neglecting time derivatives in [6] we see that ω is constant and neglecting them in [5] we can take $\bar{b} \equiv 0$ to satisfy the last equation. The third equation of [5] becomes algebraic and is used to eliminate b from the first two of [5] to one order in α. These two remaining equations and the survivor of [6] become

[7] $$a_{xx} + a_{yy} - \left[\kappa^2 - \frac{\omega^2(\omega_0^2 + \omega_p^2 - \omega^2)}{c^2(\omega_0^2 - \omega^2)}\right]a + \frac{3\alpha\omega^2\varepsilon_0^2\omega_p^6 a^3}{4c^2(\omega_0^2 - \omega^2)^4} = 0$$

[8] $$(a^2\kappa_1)_x + (a^2\kappa_2)_y = 0$$

[9] $$\kappa_{1y} - \kappa_{2x} = 0 .$$

Equations of this structure have been seen before. Cumberbatch (2) and Shvartsburg (3) derived them for a non-inertial medium and hence their interpretation of [7] differs from ours. In addition Shvartsburg used an exponential rather than cubic nonlinearity. We emphasize that the only approximation so far is the neglect of higher powers of α and while a, κ_1 and κ_2 must be slowly varying we have not yet neglected any of their derivatives.

We can make some observations immediately by solving [7] for the phase velocity ω/κ , obtaining

$$\frac{\omega}{\kappa} = \left[\frac{\omega_0^2 + \omega_p^2 - \omega^2}{c^2(\omega_0^2 - \omega^2)} + \frac{a_{xx} + a_{yy}}{\omega^2 a} + \frac{3\alpha\varepsilon_0^2\,\omega_p^6\,a^2}{4c^2(\omega_0^2 - \omega^2)^4} \right]^{-1/2} .$$

Neglecting the second derivatives and the term in α we have the phase velocity of a uniform plane wave in a linear medium. If $\omega_p = 0$ the medium becomes uncoupled from the electromagnetic wave and the wave passes at speed c . With coupling due to finite ω_p its speed is less than c and depends on the wave frequency ω . This is called a dispersive medium because the group velocity $\partial\omega/\partial\kappa$ depends on ω so that wave packets of different frequencies separate. When $\omega = \omega_0$ the wave resonates with the medium and cannot pass through it. Also there is a frequency band $\omega_0 \leq \omega \leq \sqrt{\omega_0^2 + \omega_p^2}$ where the wave reflects backwards from the medium, the amplitude falling exponentially as the wave penetrates.

When the second derivatives of amplitude become large relative to the amplitude there is an additional influence on the phase velocity. These derivatives are called higher dispersion terms since they modify the previously discussed dispersive effects and they are also responsible for diffraction. Near the peak of a local maximum in amplitude we have $a_{yy} < 0$. This negative term increases the phase velocity relative to other portions of the wave front giving a defocussing effect. Finally the term containing α gives the nonlinear influence of the medium on the wave. For $\alpha > 0$ this term causes the phase velocity to fall as amplitude rises so that a local maximum in amplitude tends to focus. When

diffraction and nonlinearity are in perfect balance we obtain the self-trapped beam of Chiao et al (4) where rays are straight.

To prepare for solving the equations we introduce the scaled variables

$$x' = Fx \ , \quad y' = Fy \ , \quad \kappa' = F^{-1}\kappa$$

$$a' = \left[\frac{3\alpha\varepsilon_0^2 \omega_p^6}{8(\omega_0^2 - \omega^2)^3 (\omega_0^2 + \omega_p^2 - \omega^2)} \right]^{1/2} a$$

where

$$F = \frac{\omega(\omega_0^2 + \omega_p^2 - \omega^2)^{1/2}}{c(\omega_0^2 - \omega^2)^{1/2}} \ .$$

Then, if the primes are dropped, [8] and [9] do not change form while [7] becomes

[10] $$a_{xx} + a_{yy} - (\kappa^2 - 1)a + 2a^3 = 0 \ .$$

2. Ray Co-ordinates and Geometrical Optics

We wish to investigate solutions to the system of equations [8], [9], [10]. In this section we consider as negligible the terms a_{xx} and a_{yy} and call this the geometrical optics approximation. Then in Section 3 we treat these higher dispersion terms equally with the other terms. Equations [8] and [9] suggest, by their similarity to the equations of conservation of mass and null vorticity in fluid flow problems, the introduction of the potentials

$$\text{[11]} \qquad \begin{aligned} \xi_x &= \kappa_1 , & \xi_y &= \kappa_2 \\ \eta_x &= -a^2\kappa_2 , & \eta_y &= a^2\kappa_1 \end{aligned}$$

since those equations would be satisfied identically. Instead of eliminating a and $\vec{\kappa}$ in favour of ξ and η, however, we use ξ and η as new independent variables. Then the vectors along co-ordinate lines,

$$\nabla\xi = (\kappa_1,\kappa_2) , \qquad \nabla\eta = a^2(-\kappa_2,\kappa_1)$$

are perpendicular so that the new co-ordinates are orthogonal. Equation [4] indicates that for fixed t, ξ and θ differ by a constant, hence the curves of constant ξ are surfaces of constant phase while the curves of constant η point in the direction of $\vec{\kappa}$ and these are the rays. To obtain a unique set of dependent variables we set

$$\kappa_1 = \kappa \cos\phi , \qquad \kappa_2 = \kappa \sin\phi$$

where ϕ is the angle rays make with the x-axis. Then x and y derivatives in [8] and [9] are converted to ξ and η derivatives with κ,ϕ,a as dependent variables . The results are

$$\text{[12]} \qquad \phi_\xi = \frac{a^2}{\kappa}\kappa_\eta , \qquad \phi_\eta = -\frac{1}{a^4\kappa}(\kappa a^2)_\xi .$$

These equations are coupled with [10] which becomes

$$\text{[13]} \qquad a\kappa^2\left[\frac{1}{3}(a^3)_{\eta\eta} - (a^{-1})_{\xi\xi}\right] - \kappa^2 + 1 + 2a^2 = 0 .$$

This equation is used to eliminate κ from [12].

It now becomes clear how the term geometrical optics applies. In linear optics the last term of [13] is missing and if the second derivatives are negligible as well then $\kappa=1$. Placing this into [12] we have $\phi_\xi = 0$ so that the orientation of a ray does not change with distance along the ray. Straight rays lead to a geometrical analysis. The term geometrical optics approximation is still used (see [1] for example) to refer to the neglect of second derivative terms in the nonlinear case even though it does not yield straight rays. Also the second derivative terms are still referred to as the diffraction terms (as in [1]) since they cause rays to bend even though the nonlinear term also causes bending of rays.

Neglecting the diffraction terms of [13] we eliminate κ from [12] obtaining

$$\phi_\xi = \frac{2a^3 a_\eta}{1+2a^2} , \qquad \phi_\eta = -\frac{2}{a^3}\left(\frac{1+3a^2}{1+2a^2}\right) a_\xi .$$

We are able to notice an exact solution by interchanging the roles of dependent and independent variables by a hodograph transformation. It is convenient to set $\rho = a^2$ and then we have

$$\eta_\rho = \frac{\rho}{1+2\rho}\,\xi_\phi , \qquad \xi_\rho = -\frac{1+3\rho}{\rho^2(1+2\rho)}\,\eta_\phi .$$

One can now eliminate ξ or η obtaining a separable equation in the other. A solution that represents a focussing beam is

$$\text{[14]} \qquad \eta = -\frac{\sin\phi}{\sqrt{1+2\rho}} , \qquad \xi = -\frac{\cos\phi}{\rho\sqrt{1+2\rho}} .$$

The solution written relative to x and y is found by inverting [11] and using [13] (with diffraction neglected) for κ. This inversion gives x and y as line integrals in the variables ξ and η and then [14] converts these to line integrals in ϕ and ρ. With the focal point placed at the origin to fix integration constants, integration gives

$$x = -\frac{1}{2\rho} - \log\frac{2\rho}{1+2\rho} - \frac{\cos 2\phi}{2\rho(1+2\rho)}$$

[15]

$$y = -\frac{\sin 2\phi}{2\rho(1+2\rho)} \; .$$

Equations [15] implicitly determine the amplitude ρ and the ray orientation ϕ for a suitable domain in x and y. We establish the positions of rays by fixing η. Setting $\eta = \bar{\eta}$ in the first equation of [14], ϕ is eliminated from [15] giving rays identified by $\bar{\eta}$ and parameterized by ρ. The resulting equations are

$$x = \frac{\bar{\eta}^2}{\rho} - \frac{1+\rho}{\rho(1+2\rho)} - \log\frac{2\rho}{1+2\rho}$$

$$y = \left(\frac{1-(1+2\rho)\bar{\eta}^2}{1+2\rho}\right)^{1/2} \frac{\bar{\eta}}{\rho} \; .$$

Similarly one can obtain a parameterized form of the surfaces of constant phase by fixing $\xi = \bar{\xi}$ in the second equation of [14] and eliminating ϕ from [15]. The rays are plotted in Figure 2a. If ϕ is eliminated from [15] one can obtain profiles of the beam at various positions x and these are plotted in Figure 2b.

Solution [15] appears in (5) and further focussing solutions appear in (6). A difficulty with these solutions is that in the region of the focus, equations [15] fail to apply since the expansion for small α has broken down. If indeed there is a focus where the amplitude becomes large or singular, substitution [3] is unsuitable because the oscillations will not be close to sinusoidal. In fact uniform plane wave solutions to [1] and [2] take the form of Jacobian elliptic functions. Clearly [15] cannot be correct globally but since the only indication of trouble is in the focal region we would hope that [15] holds outside the focal region. Also we would hope that it is the geometrical optics approximation that permits the singularity to occur and that retaining the diffraction terms may prevent the focus. This is investigated in the next section.

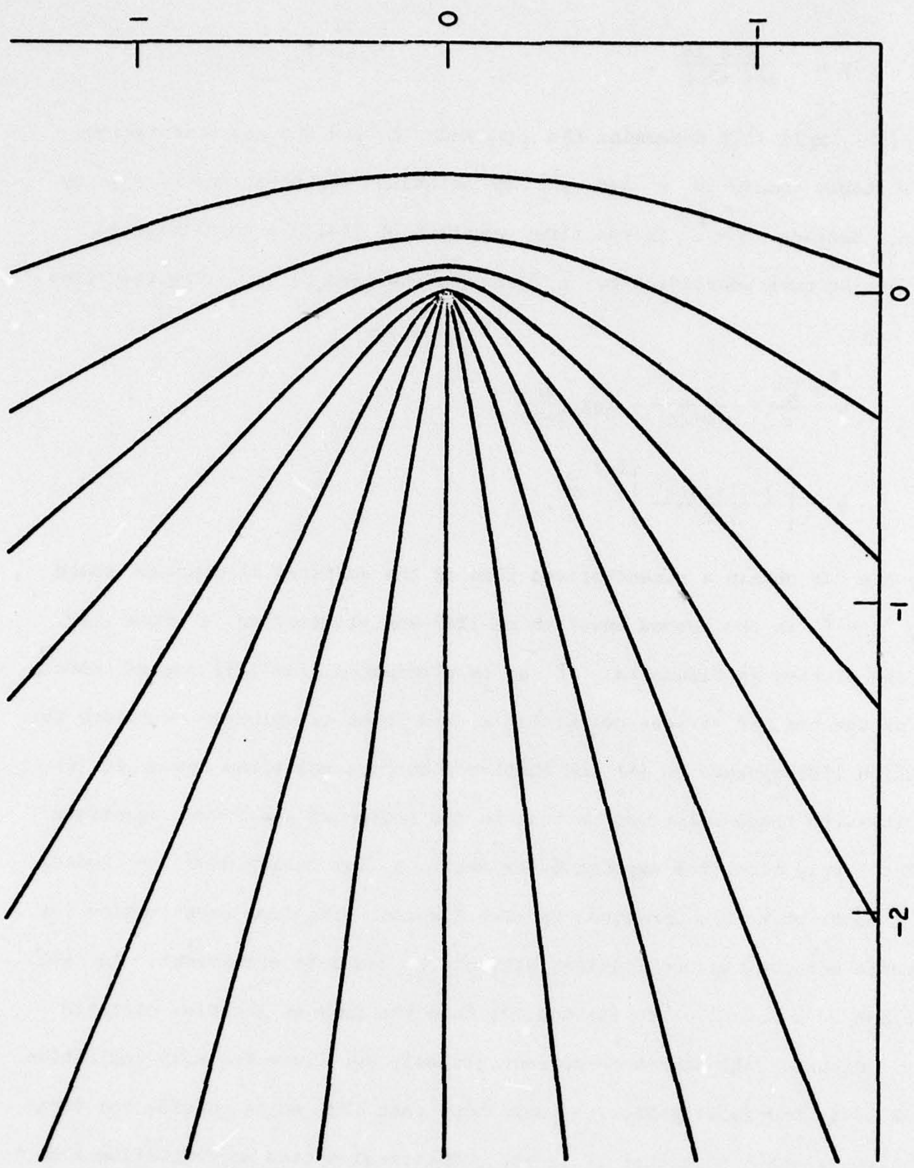

Figure 2a. Ray Pattern for a Focussing Solution

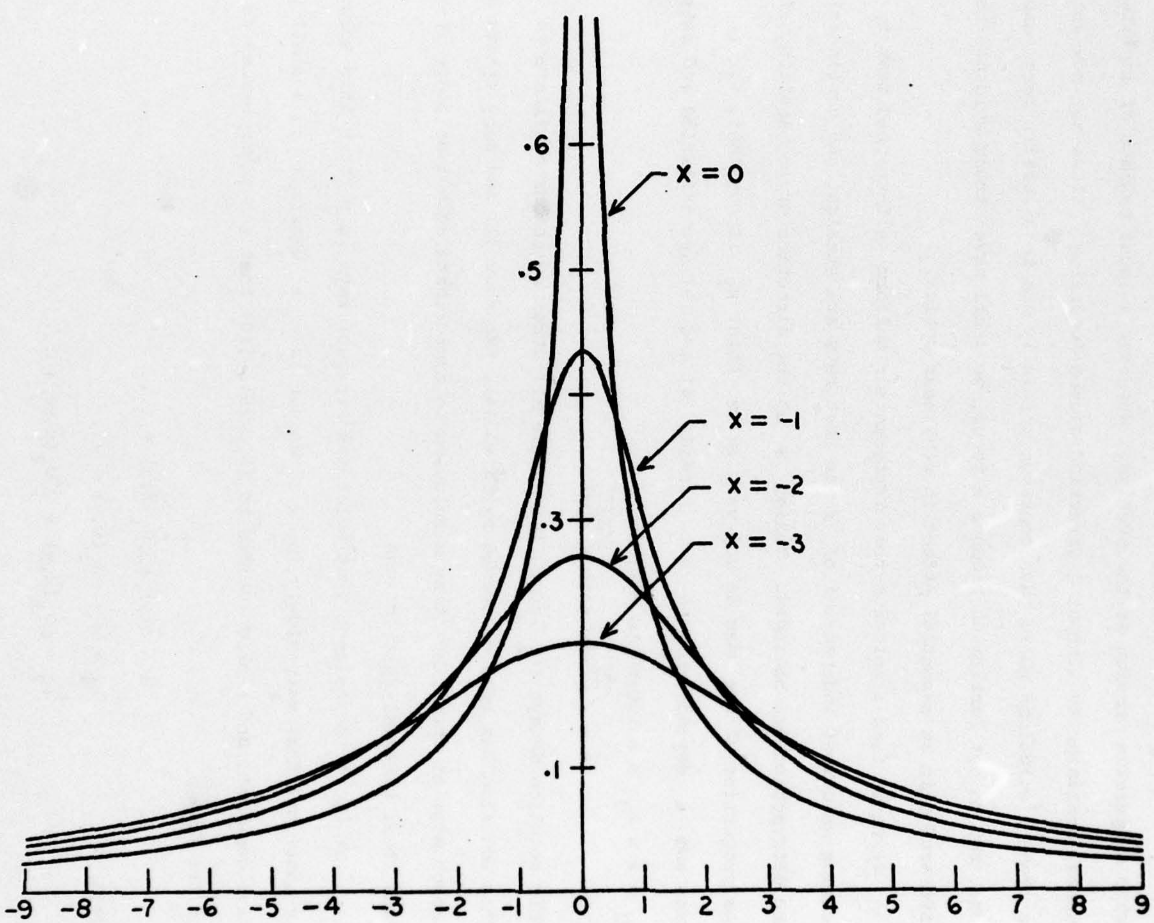

Figure 2b. Amplitude Profiles for a Focussing Solution

3. Self-Trapped Beams

Having noted that singularities occur in the geometrical optics approximation we return to equations [8], [9], [10] to investigate solutions to this fully dispersive version of the problem. Whenever a local maximum of amplitude occurs, nonlinearity without diffraction causes focussing. If we are now able to produce solutions where rays converge initially due to a locally large amplitude, but do not further on produce a focus, we shall have strong evidence that diffraction is an essential effect in nonlinear optics.

We first find a solution that represents a uniform self-trapped beam by seeking solutions independent of x so that rays are straight and nonlinearity and diffraction are balanced. Taking x as the direction of propagation, if the properties of the beam do not vary in x then κ_1 is constant, κ_2 is zero and a depends only on y. Already [8] and [9] are satisfied and setting $\kappa_1 = \bar{\kappa} > \kappa_0$, a solution to [10] is

$$\bar{a} = \sqrt{\bar{\kappa}^2 - 1} \operatorname{sech} \sqrt{\bar{\kappa}^2 - 1}\, y \, . \tag{16}$$

This solution decays for large $|y|$ while all other solutions oscillate in y. This solution has been discussed by Chiao (4), Akhmanov (1) and many others although most of them find it as a solution to approximate equations since it satisfies their paraxial restrictions.

To upset the balance that maintains straight rays we perturb this solution by functions that vary slowly in x. We set $\bar{x} = \varepsilon x$, where ε is a small positive constant, and seek solutions to [8], [9], [10] that are power series in ε of the form

$$\begin{aligned} a &= \bar{a}(y) + \varepsilon G_1(\bar{x},y) + \ldots \\ \kappa_1 &= \bar{\kappa} + \varepsilon U_1(\bar{x},y) + \ldots \\ \kappa_2 &= \varepsilon V_1(\bar{x},y) + \varepsilon^2 V_2(\bar{x},y) + \ldots \end{aligned} \tag{17}$$

As it happens this expansion gives acceptable solutions for the quantities displayed but to extend to higher accuracy the perturbations have a small effect on the original solution $\bar{a}(y)$. This is accounted for by replacing y by the new variable $\bar{y} = (1 + \varepsilon^2\gamma_2 + \varepsilon^3\gamma_3 + \ldots)y$ where the γ_i are determined by suppressing secular terms in the G_i at each stage of solution. For simplicity we shall only seek beams that are symmetric about the x-axis. This requires that G_1 and U_1 be even in y and that V_1 and V_2 be odd in y.

Placing [17] into [8], [9], [10] and equating coefficients of like powers of ε we obtain a hierarchy of equations. From [8] we obtain an equation that determines V_1 to be

$$V_1 = C \cosh^2 \sqrt{\bar{\kappa}^2-1}\, y$$

and the equation determining V_2 is

[18] $$(\bar{a}^2V_2)_y + (2\bar{a}G_1V_1)_y + 2\bar{a}\bar{\kappa}G_{1\bar{x}} + \bar{a}^2U_{1\bar{x}} = 0 \,.$$

From [9] we find that U_1 must be a function of $\bar{x}$, hence we set

$$U_1 = u(\bar{x}) \,.$$

Finally [10] gives the equation determining G_1 as

[19] $$G_{1yy} - (\bar{\kappa}^2-1)G_1 - 2\bar{\kappa}\bar{a}U_1 + 6\bar{a}^2G_1 = 0 \,.$$

Since V_1 must be odd we take $C = 0$ and this necessitates finding V_2 to obtain the y-component of $\vec{\kappa}$. While U_1 is an arbitrary function of $\bar{x}$, succeeding terms in expansion [17] will not be small unless $u(\bar{x})$ and all its derivatives are slowly varying. Hence $u(\bar{x})$ must be analytic in $\bar{x}$. When U_1 is placed into [19] the $\bar{x}$-dependence factors out. Setting $G_1 = u(\bar{x})g(y)$, [19] becomes

$$g_{yy} - (\bar{\kappa}^2-1)(1-6\,\mathrm{sech}^2\sqrt{\bar{\kappa}^2-1}\,y)g = 2\bar{\kappa}\sqrt{\bar{\kappa}^2-1}\,\mathrm{sech}\sqrt{\bar{\kappa}^2-1}\,y \,.$$

The bounded solution of this equation is found by differentiating [16] with respect to $\bar{\kappa}$ and y since perturbing $\bar{\kappa}$ by a constant gives another solution of the form [16]. Thus we obtain

[20] $$G_1 = u(\bar{x})\left[\frac{\bar{\kappa}}{\sqrt{\bar{\kappa}^2-1}} \operatorname{sech} \sqrt{\bar{\kappa}^2-1}\, y - \bar{\kappa}\, y \operatorname{sech} \sqrt{\bar{\kappa}^2-1}\, y \tanh \sqrt{\bar{\kappa}^2-1}\, y \right.$$

$$\left. + A \operatorname{sech} \sqrt{\bar{\kappa}^2-1}\, y \tanh \sqrt{\bar{\kappa}^2-1}\, y \right] .$$

Since G_1 must be even in y we take $A = 0$. Then we solve [18] for V_2 obtaining

[21] $$V_2 = - \frac{1}{\bar{\kappa}^2-1} \frac{du(\bar{x})}{d\bar{x}} \left[\bar{\kappa}^2 y + \frac{2\bar{\kappa}^2-1}{2(\bar{\kappa}^2-1)^{1/2}} \sinh 2\sqrt{\bar{\kappa}^2-1}\, y \right] .$$

The perturbation is now fixed when the arbitrary function $u(\bar{x})$ is specified.

To obtain the beam of Figure 1b we could take $u(\bar{x}) = \cos \bar{x}$, for example, to have a beam with periodically converging and diverging rays but no focus. Alternatively we could take $u(\bar{x}) = \bar{x}^{-1}$ to have a beam that does focus. Thus we assert that a focus may or may not result when waves in a nonlinear focussing medium begin to converge and that diffraction is sufficient to prevent focussing although it does not necessarily do so.

REFERENCES

(1) S. A. Akhmanov, A. P. Sukhorukov and R. V. Khokhlov, Soviet Physics, J. E. T. P., 23 , 1025, (1966).

(2) E. Cumberbatch, J. Inst. Maths. Applics., 6 , 250, (1970).

(3) A. B. Shvartsburg, Sov. Phys. Doklady, 16, 9, (1972).

(4) R. Y. Chiao, E. Garmire and C. H. Townes, Phys. Rev. Letters, 13, 479, (1964).

(5) M. N. Y. Anwar, Ph.D. Thesis, University of New Brunswick, 1978.

(6) R. D. Small, Ph.D. Thesis, California Institute of Technology, 1973.

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